

**Supplementary Table 2.** T1-weighted spherical mean diffusion tensor (T1-SMDT) model fitting based on artificial neural networks.

<u>Signal model</u>	$s(b, \text{TI}, \text{TS}) = \frac{\sqrt{\pi}}{2} s_0 \left  1 - e^{-\frac{\text{TI}}{T_1}} - \left( 1 - e^{-\frac{\text{TS}}{T_1}} \right) e^{-\frac{\text{TI}}{T_1}} \right  e^{-bd_{\perp}} \frac{\text{erf}\left(\frac{\sqrt{b(d_{\parallel} - d_{\perp})}}{\sqrt{b(d_{\parallel} - d_{\perp})}}\right)}{\sqrt{b(d_{\parallel} - d_{\perp})}}$ <p><i>b</i>: b-value; TS: saturation time (delay saturation-inversion); TI: inversion time (delay inversion-excitation)</p>
<u>Tissue parameters</u>	<p><math>s_0</math> (apparent proton density): fitting range [0.5; 5.0]  <math>T_1</math> (longitudinal relaxation time): fitting range [100; 4000] ms  <math>d_{\parallel}</math> (parallel fibre diffusivity): fitting range [0.01; 3.20] <math>\mu\text{m}^2 \text{ms}^{-1}</math>  <math>d_{\perp}</math> (perpendicular fibre diffusivity): parametrised as <math>d_{\perp} = k d_{\parallel}</math>, fitting range for <math>k</math> [0.00; 0.99]</p>
<u>DNN implementation</u>	<ul style="list-style-type: none"> <li>The DNN is built of a cascade of hidden layers (here 7), each consisting of a linear matrix operation followed by ReLU activation, with <math>\text{ReLU}(x) = \max(0, x)</math></li> <li>The DNN features <math>N</math> input neurons (as many as the number of input measurements per voxel) and 4 output neurons, which map the 4 tissue parameters: <ul style="list-style-type: none"> <li>for fully-sampled signals we use {32, 28, 24, 20, 16, 12, 8, 4} neurons;</li> <li>for 16-measurement sub-protocols we use {16, 14, 13, 11, 9, 7, 6, 4} neurons;</li> <li>for 8-measurement sub-protocols we use {8, 7, 7, 6, 6, 5, 5, 4} neurons;</li> <li>for 4-measurement sub-protocols we use {4, 4, 4, 4, 4, 4, 4, 4} neurons</li> </ul> </li> <li>The <math>i</math>-th output neuron activation <math>u_i</math>, defined in the range <math>0 \leq u_i \leq u_{max}</math>, is mapped to the <math>i</math>-th tissue parameter <math>p_i</math> as <math display="block">p_i = \frac{2}{1 + e^{-\alpha_i (\log(\text{softplus}(u_i)) - \log(\log(2)))}} - 1,</math> <p>where <math>\alpha_i</math> is a learnable scaling factor and <math>\text{softplus}(x) = \log(1 + e^x)</math></p> </li> <li>Output MRI signals are calculated from tissue parameters <math>p_i</math> with the equation above</li> </ul>
<u>DNN training</u>	<ul style="list-style-type: none"> <li>Input measurements <math>\{s(b_n, \text{TI}_n, \text{TS}_n) \mid n = 1, \dots, N\}</math> from each voxel are normalised by computing <math>a(b_n, \text{TI}_n, \text{TS}_n) = s(b_n, \text{TI}_n, \text{TS}_n) / \max_n (s(b_n, \text{TI}_n, \text{TS}_n))</math></li> <li>The DNN is optimised by backpropagating the <math>\ell^2</math>-norm of the error (i.e. mean squared error, MSE) between ground truth MRI measurements and signal prediction</li> <li>Optimisation is performed with ADAM on synthetic MRI signals for 50 epochs (learning rate of <math>10^{-4}</math>; one update per mini-batch of 100 voxels) for sub-protocols, repeating the training 10 times with different random DNN initialisations</li> <li>Synthetic MRI signals are computed for uniformly distributed tissue parameters within the ranges reported above, adding Rician noise with <math>\text{SNR} = \frac{s_0}{\sigma_{\text{noise}}}</math> within the range [10; 100]. We use 80,000 voxels as training set and 20,000 as validation set. Note that parameter <math>s_0</math> in the model T1-SMDT is strongly T2-weighted (unlike in the HM-MRI model): as a consequence, SNR calculated with respect to <math>s_0</math> is expected to be lower in T1-SMDT than HM-MRI for the same <math>\sigma_{\text{noise}}</math></li> <li>The DNN providing the minimum validation loss is deployed</li> </ul>