

Supplementary Table 4. Hybrid-multidimensional MRI (HM-MRI) model fitting based on artificial neural networks.

<u>Signal model</u>	$s(b, TE) = s_0 \left(v_l e^{-b d_l - \frac{TE}{T_{2l}}} + (1 - v_l) \left(v_e e^{-b d_e - \frac{TE}{T_{2e}}} + (1 - v_e) e^{-b d_s - \frac{TE}{T_{2s}}} \right) \right)$ <p style="text-align: center;">b: b-value; TE: echo time</p>
<u>Tissue parameters</u>	<p>s_0 (apparent proton density): fitting range [0.5; 10.0] v_l (luminal water voxel volume fraction): fitting range [0.0; 1.0] v_e (epithelial fraction of non-luminal tissue): fitting range [0.0; 1.0] d_l (luminal water diffusivity): set to $2.5 \mu\text{m}^2 \text{ms}^{-1}$ d_e (epithelial water diffusivity): set to $0.4 \mu\text{m}^2 \text{ms}^{-1}$ d_s (stromal water diffusivity): set to $1.4 \mu\text{m}^2 \text{ms}^{-1}$ T_{2l} (luminal water T2): set to 650 ms T_{2e} (epithelial water T2): set to 45 ms T_{2s} (stromal water T2): set to 95 ms</p>
<u>DNN implementation</u>	<ul style="list-style-type: none"> The DNN is built of a cascade of hidden layers (here 7), each consisting of a linear matrix operation followed by ReLU activation, with $\text{ReLU}(x) = \max(0, x)$ The DNN features N input neurons (as many as the number of input measurements per voxel) and 9 output neurons, which map the 9 tissue parameters: <ul style="list-style-type: none"> for fully-sampled signals we use {16, 15, 14, 13, 12, 11, 10, 9} neurons; for 12-measurement sub-protocols we use {12, 12, 11, 11, 10, 10, 9, 9} neurons; for 9-measurement sub-protocols we use {9, 9, 9, 9, 9, 9, 9, 9} neurons The i-th output neuron activation u_i, defined in the range $0 \leq u_i \leq u_{max}$, is mapped to the i-th tissue parameter p_i as $p_i = \frac{2}{1 + e^{-\alpha_i (\log(\text{softplus}(u_i)) - \log(\log(2)))}} - 1,$ where α_i is a learnable scaling factor and $\text{softplus}(x) = \log(1 + e^x)$ Output MRI signals are calculated from tissue parameters p_i with the equation above
<u>DNN training</u>	<ul style="list-style-type: none"> Input measurements $\{s(b_n, TE_n) \mid n = 1, \dots, N\}$ from each voxel are normalised by computing $a(b_n, TE_n) = s(b_n, TE_n) / \max_n (s(b_n, TE_n))$ The DNN is optimised by backpropagating the ℓ^2-norm of the error (i.e. mean squared error, MSE) between ground truth MRI measurements and signal prediction Optimisation is performed with ADAM on synthetic MRI signals for 50 epochs (learning rate of 10^{-4}; one update per mini-batch of 100 voxels) for sub-protocols, repeating the training 10 times with different random DNN initialisations Synthetic MRI signals are computed for uniformly distributed tissue parameters within the ranges reported above, adding Rician noise with $SNR = \frac{s_0}{\sigma_{\text{noise}}}$ within the range [35; 250]. We use 80,000 voxels as training set and 20,000 as validation set The DNN providing the minimum validation loss is deployed