

**Supplementary Table 4.** Hybrid-multidimensional MRI (HM-MRI) model fitting based on artificial neural networks.

<u>Signal model</u>	$s(b, TE) = s_0 \left( v_l e^{-b d_l - \frac{TE}{T_{2l}}} + (1 - v_l) \left( v_e e^{-b d_e - \frac{TE}{T_{2e}}} + (1 - v_e) e^{-b d_s - \frac{TE}{T_{2s}}} \right) \right)$ <p style="text-align: center;"><math>b</math>: b-value; TE: echo time</p>
<u>Tissue parameters</u>	<p><math>s_0</math> (apparent proton density): fitting range [0.5; 10.0]  <math>v_l</math> (luminal water voxel volume fraction): fitting range [0.0; 1.0]  <math>v_e</math> (epithelial fraction of non-luminal tissue): fitting range [0.0; 1.0]  <math>d_l</math> (luminal water diffusivity): set to <math>2.5 \mu\text{m}^2 \text{ms}^{-1}</math>  <math>d_e</math> (epithelial water diffusivity): set to <math>0.4 \mu\text{m}^2 \text{ms}^{-1}</math>  <math>d_s</math> (stromal water diffusivity): set to <math>1.4 \mu\text{m}^2 \text{ms}^{-1}</math>  <math>T_{2l}</math> (luminal water T2): set to 650 ms  <math>T_{2e}</math> (epithelial water T2): set to 45 ms  <math>T_{2s}</math> (stromal water T2): set to 95 ms</p>
<u>DNN implementation</u>	<ul style="list-style-type: none"> <li>The DNN is built of a cascade of hidden layers (here 7), each consisting of a linear matrix operation followed by ReLU activation, with <math>\text{ReLU}(x) = \max(0, x)</math></li> <li>The DNN features <math>N</math> input neurons (as many as the number of input measurements per voxel) and 9 output neurons, which map the 9 tissue parameters: <ul style="list-style-type: none"> <li>for fully-sampled signals we use {16, 15, 14, 13, 12, 11, 10, 9} neurons;</li> <li>for 12-measurement sub-protocols we use {12, 12, 11, 11, 10, 10, 9, 9} neurons;</li> <li>for 9-measurement sub-protocols we use {9, 9, 9, 9, 9, 9, 9, 9} neurons</li> </ul> </li> <li>The <math>i</math>-th output neuron activation <math>u_i</math>, defined in the range <math>0 \leq u_i \leq u_{max}</math>, is mapped to the <math>i</math>-th tissue parameter <math>p_i</math> as <math display="block">p_i = \frac{2}{1 + e^{-\alpha_i (\log(\text{softplus}(u_i)) - \log(\log(2)))}} - 1,</math> where <math>\alpha_i</math> is a learnable scaling factor and <math>\text{softplus}(x) = \log(1 + e^x)</math> </li> <li>Output MRI signals are calculated from tissue parameters <math>p_i</math> with the equation above</li> </ul>
<u>DNN training</u>	<ul style="list-style-type: none"> <li>Input measurements <math>\{s(b_n, TE_n) \mid n = 1, \dots, N\}</math> from each voxel are normalised by computing <math>a(b_n, TE_n) = s(b_n, TE_n) / \max_n (s(b_n, TE_n))</math></li> <li>The DNN is optimised by backpropagating the <math>\ell^2</math>-norm of the error (i.e. mean squared error, MSE) between ground truth MRI measurements and signal prediction</li> <li>Optimisation is performed with ADAM on synthetic MRI signals for 50 epochs (learning rate of <math>10^{-4}</math>; one update per mini-batch of 100 voxels) for sub-protocols, repeating the training 10 times with different random DNN initialisations</li> <li>Synthetic MRI signals are computed for uniformly distributed tissue parameters within the ranges reported above, adding Rician noise with <math>SNR = \frac{s_0}{\sigma_{\text{noise}}}</math> within the range [35; 250]. We use 80,000 voxels as training set and 20,000 as validation set</li> <li>The DNN providing the minimum validation loss is deployed</li> </ul>